

# Scaling Properties of Traffic in Communication Networks

---

## Probabilistic Resources Allocation in Cloud Environments

**Paulo Gonçalves**

Inria (DANTE) - ENS Lyon

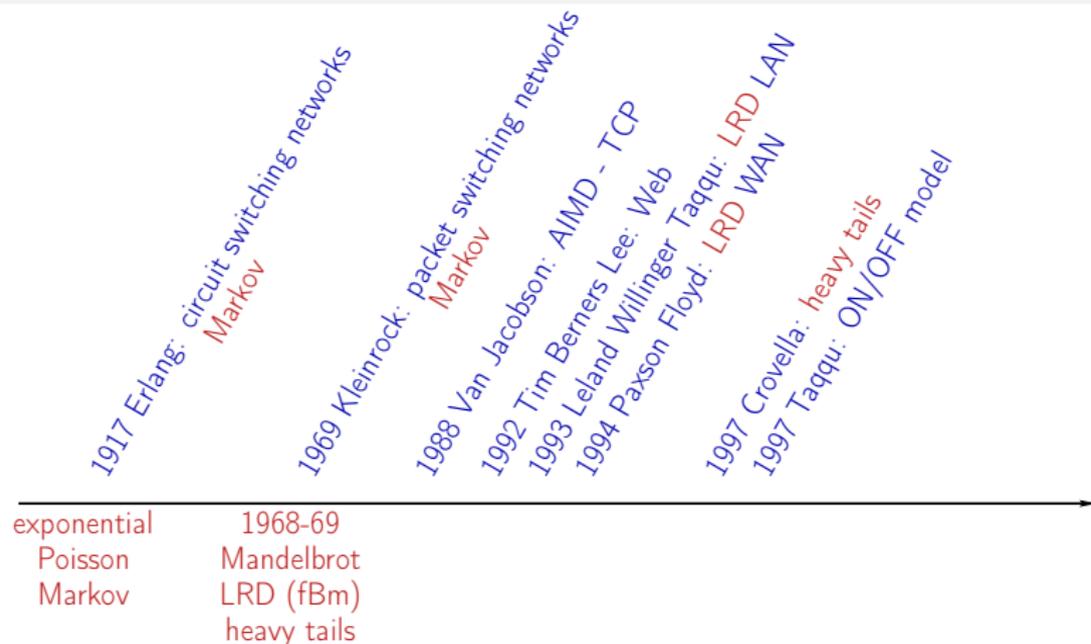
Shubhabrata Roy (PhD, 2010-2013)

Thomas Begin (Ass. Prof., UCBL Lyon 1)

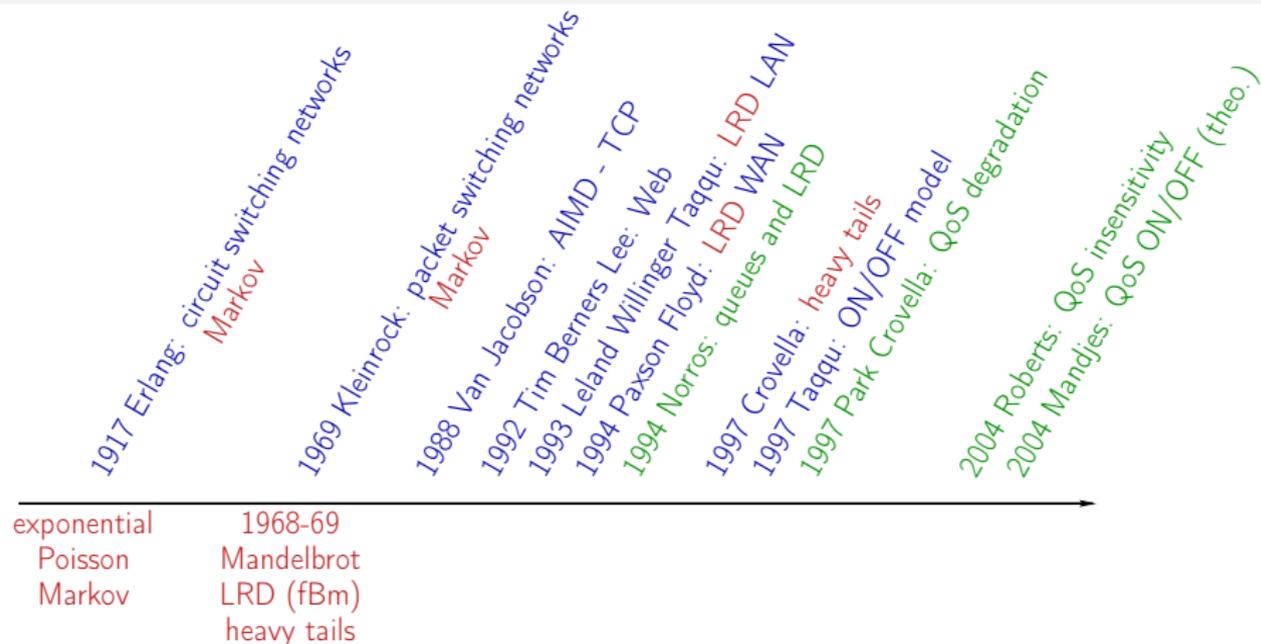
Patrick Loiseau (Ass. Prof. EURECOM)

**CLOSER 2014 – Barcelona (Spain) 3-5 April 2014**

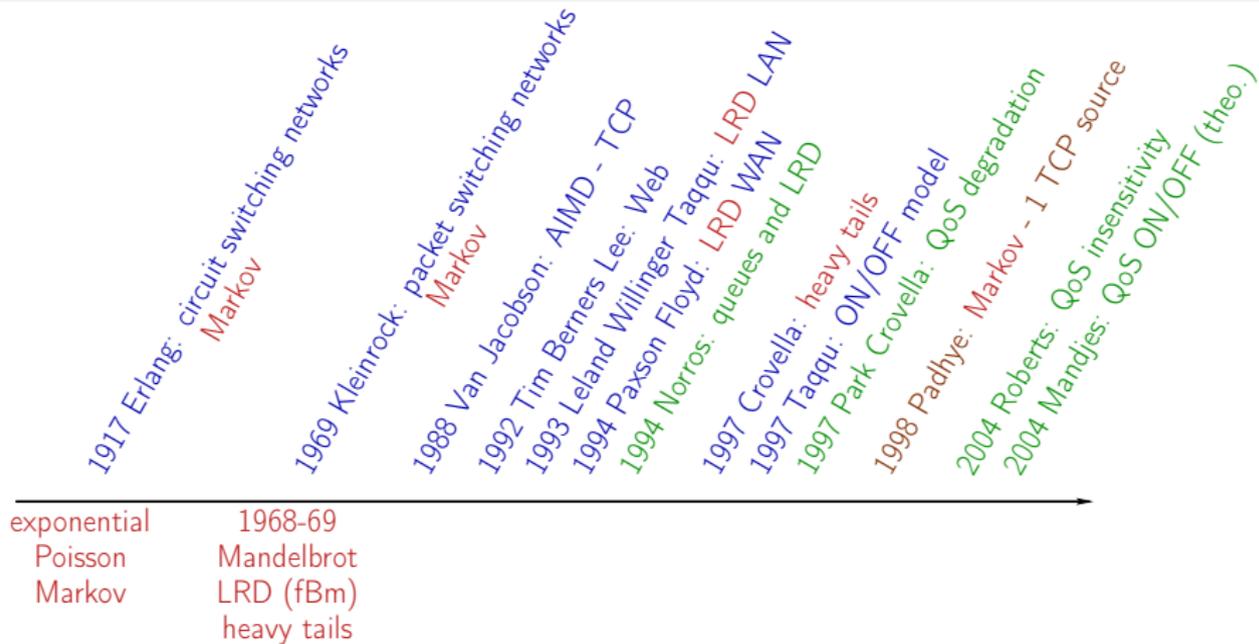
# Historical perspective



# Historical perspective



# Historical perspective



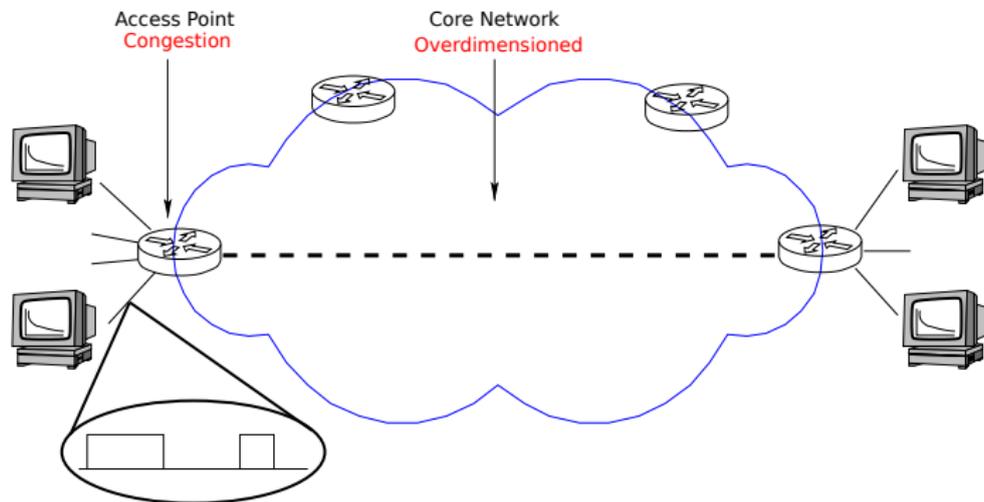
Some open questions:

- Long Range Dependence / Heavy Tailed distributions impact on QoS ?
- Existing models (e.g. Padhye) only predict mean metrics (e.g. throughput) : what about variability?

# Our approach

To combine **theoretical models** with **controlled experiments** in realistic environments and **real-world traffic traces**

## Simplified System



- Congestion essentially arises at the access points  
     → Simplified System : single bottleneck
- Users' behavior : ON/OFF source model
- *MetroFlux*: a probe for traffic capture at packet level (O. Goga, ...)

# Long memory in aggregated traffic: the Taqqu model

- Heavy-tailed distributed ON periods: **heavy tail** index  $\alpha_{ON} > 1$

## Theorem (Taqqu, Willinger, Sherman, 1997)

*In the limit of a large number of sources  $N_{src}$ , if:*

- *flow throughput is constant,*
- *same throughput for all flows ;*

*aggregated bandwidth  $B^{(\Delta)}(t)$  is long range dependent, with parameter:*

$$H = \max\left(\frac{3 - \alpha_{ON}}{2}, \frac{1}{2}\right)$$

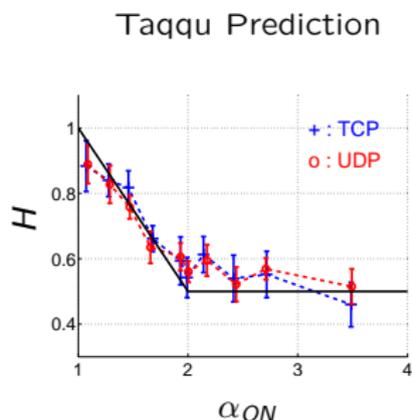
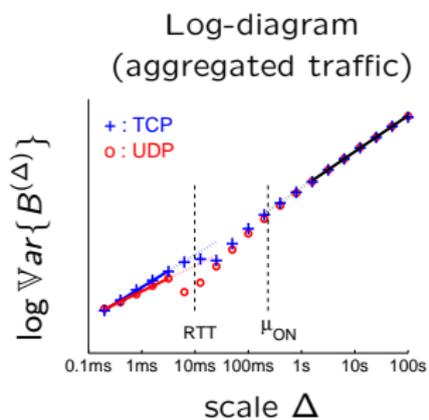
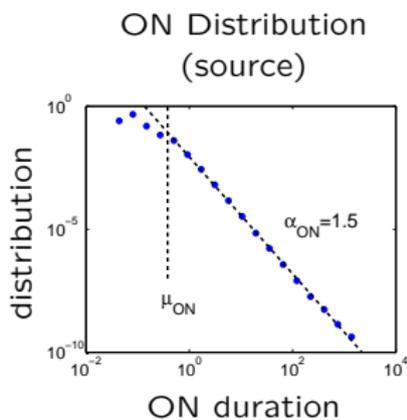
**Long memory: long range correlation** ( $H > 1/2$ )

$$\text{Cov}_{B^{(\Delta)}}(\tau) = \mathbb{E}\left\{B^{(\Delta)}(t)B^{(\Delta)}(t + \tau)\right\} \underset{\tau \rightarrow \infty}{\sim} \tau^{(2H-2)}$$

Variance grows faster than  $\Delta$ :  $\text{Var}\{B^{(\Delta)}(t)\} \sim \Delta^{2H}$

# Theorem validation on a realistic environment

- Controlled experiment: *MetroFlux* 1 Gbps, 100 sources, 8 hours traffic
- UDP/TCP: throughput limited to 5 Mbps (no congestion)

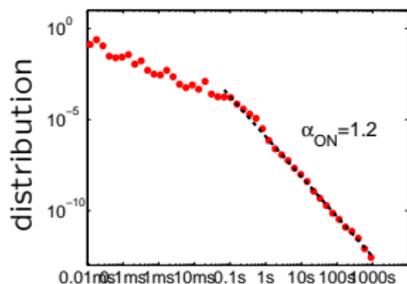


- ⇒ Protocol has no influence at large scales
- ⇒ Long memory shows up beyond scale  $\Delta = \mu_{ON}$  (mean flow duration)

# Influence of flow mean throughput / duration correlation

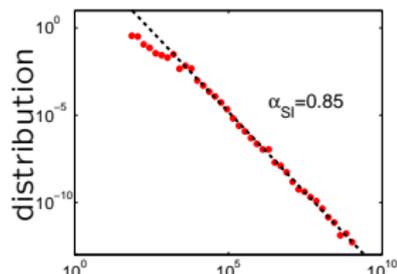
- Web traffic acquired at in2p3 (Lyon) with *MetroFlux* 10 Gbps

### ON Distribution



ON duration

### Size Distribution

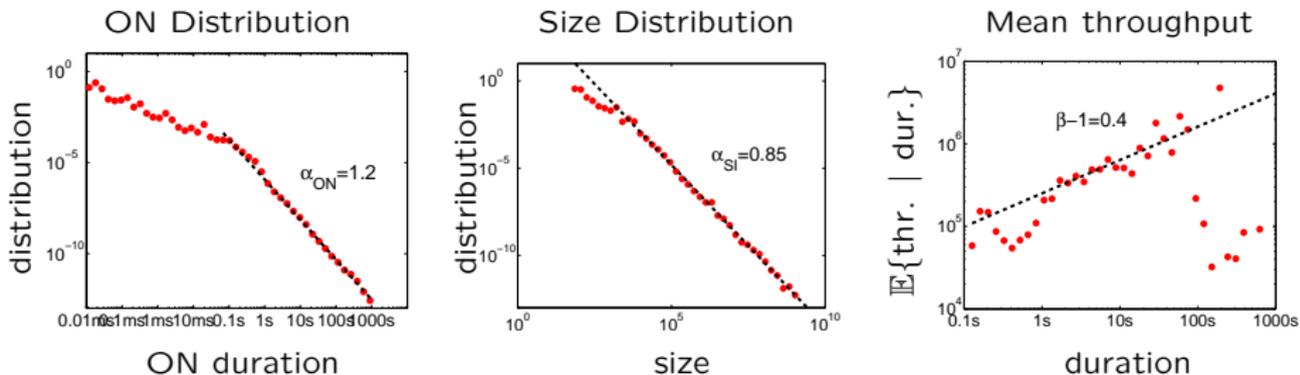


size

- Heavy-tailed ON periods,  $\alpha_{ON} = 1.2$
- Heavy tailed flow sizes,  $\alpha_{SI} = 0.85$

# Influence of flow mean throughput / duration correlation

- Web traffic acquired at in2p3 (Lyon) with *MetroFlux* 10 Gbps



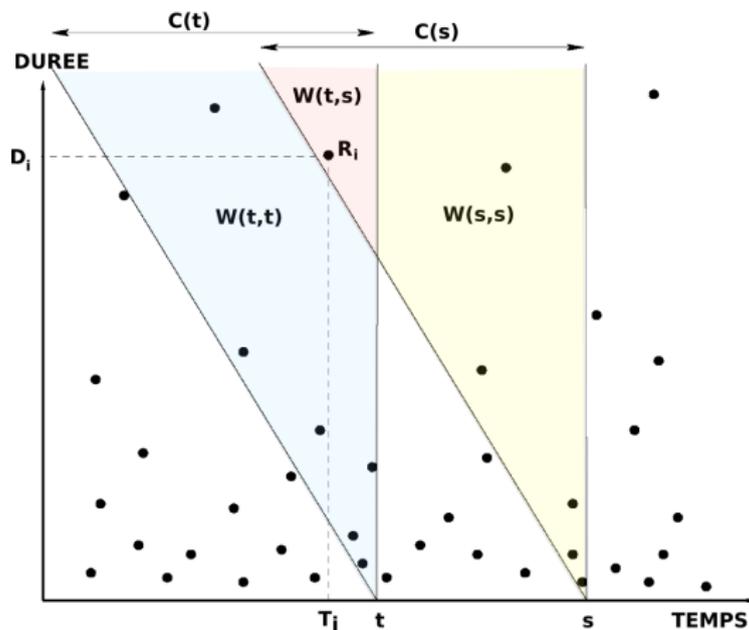
- Heavy-tailed ON periods,  $\alpha_{ON} = 1.2$
- Heavy tailed flow sizes,  $\alpha_{SI} = 0.85$
- Flow throughput and duration are correlated:

$$\mathbb{E}\{\text{thr.} | \text{dur.}\} \propto (\text{dur.})^{\beta-1}, \quad \beta = \alpha_{ON} / \alpha_{SI} (= 1.4)$$

$\Rightarrow$  Which heavy tail index does control LRD ? ( $\alpha_{ON}$ ,  $\alpha_{SI}$ ) ?

# Taqqu model extension

- Planar Poisson process to describe arrival instant vs duration



# Taqqu model extension

- Planar Poisson process to describe arrival instant vs duration

Proposition (LGVBP, 2009)

Model:  $\mathbb{E}\{\text{through.}|dur.\} = M \cdot (dur.)^{\beta-1}$ ;  $\mathbb{V}ar\{\text{through.}|dur.\} = V$

$$Cov_{B(\Delta)}(\mathcal{T}) = CM^2 \mathcal{T}^{-(\alpha_{ON}-2(\beta-1))+1} + C' V \mathcal{T}^{-\alpha_{ON}+1}$$

# Taqqu model extension

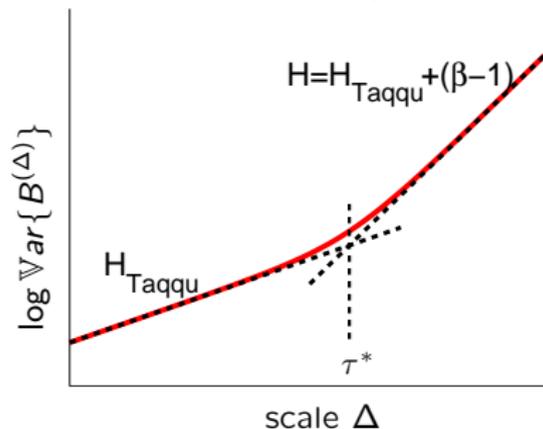
- Planar Poisson process to describe arrival instant vs duration

## Proposition (LGVBP, 2009)

Model:  $\mathbb{E}\{\text{through.}|dur.\} = M \cdot (dur.)^{\beta-1}$ ;  $\mathbb{V}ar\{\text{through.}|dur.\} = V$

$$\text{Cov}_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha_{ON}-2(\beta-1))+1} + C'V\tau^{-\alpha_{ON}+1}$$

Log-diagram,  $\beta > 1$



- threshold  $\tau^* = \left(\frac{C'V}{CM^2}\right)^{1/(2(\beta-1))}$

# Taqqu model extension

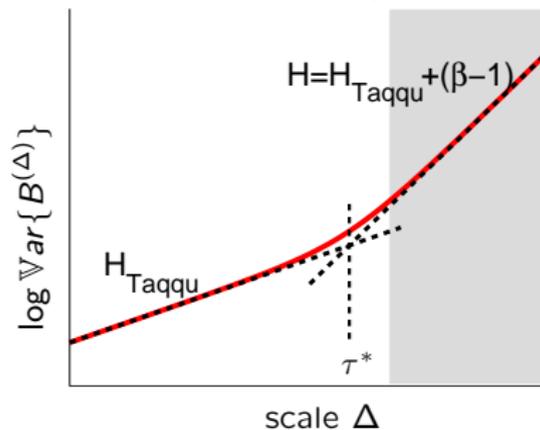
- Planar Poisson process to describe arrival instant vs duration

## Proposition (LGVBP, 2009)

Model:  $\mathbb{E}\{\text{through.}|dur.\} = M \cdot (dur.)^{\beta-1}$ ;  $\text{Var}\{\text{through.}|dur.\} = V$

$$\text{Cov}_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha_{ON}-2(\beta-1))+1} + C' V \tau^{-\alpha_{ON}+1}$$

Log-diagram,  $\beta > 1$



- threshold  $\tau^* = \left(\frac{C'V}{CM^2}\right)^{1/(2(\beta-1))}$   
 $\rightarrow$  if  $\Delta \gg \tau^*$ :  $H = H_{\text{Taqqu}} + (\beta - 1)$

# Taqqu model extension

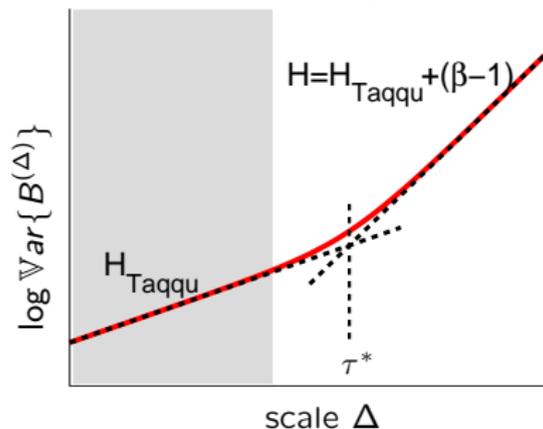
- Planar Poisson process to describe arrival instant vs duration

## Proposition (LGVBP, 2009)

Model:  $\mathbb{E}\{\text{through.}|dur.\} = M \cdot (dur.)^{\beta-1}$ ;  $\mathbb{V}ar\{\text{through.}|dur.\} = V$

$$Cov_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha_{ON}-2(\beta-1))+1} + C'V\tau^{-\alpha_{ON}+1}$$

Log-diagram,  $\beta > 1$



- threshold  $\tau^* = \left(\frac{C'V}{CM^2}\right)^{1/(2(\beta-1))}$ 
  - $\rightarrow$  if  $\Delta \gg \tau^*$ :  $H = H_{\text{Taqqu}} + (\beta - 1) \Delta^{\beta-1}$
  - $\rightarrow$  if  $\Delta \ll \tau^*$ :  $H = H_{\text{Taqqu}}$

# Taqqu model extension

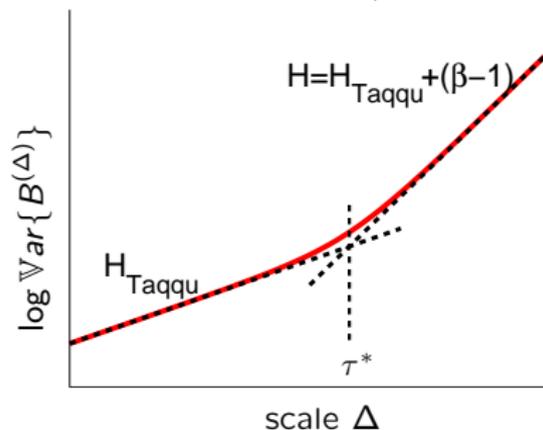
- Planar Poisson process to describe arrival instant vs duration

## Proposition (LGVBP, 2009)

Model:  $\mathbb{E}\{\text{through.}|dur.\} = M \cdot (dur.)^{\beta-1}$ ;  $\mathbb{V}ar\{\text{through.}|dur.\} = V$

$$Cov_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha_{ON}-2(\beta-1))+1} + C' V \tau^{-\alpha_{ON}+1}$$

Log-diagram,  $\beta > 1$



- Correlations intensify LRD ( $\beta > 1$ )
- Traffic evolution, future Internet: “flow-aware” control mechanisms, FTTH

# LRD impact on QoS: a brief (experimental) outlook

The situation is complex. . .

# LRD impact on QoS: a brief (experimental) outlook

The situation is complex. . .

- Negative on finite queues with UDP flows [cf. Mandjes, 2004 (infinite queues)]
  - LRD degrades QoS for large queue sizes (beyond some threshold)
  - **but** the threshold depends on the considered QoS metric (loss rate vs mean load)

# LRD impact on QoS: a brief (experimental) outlook

The situation is complex. . .

- Negative on finite queues with UDP flows [cf. Mandjes, 2004 (infinite queues)]
  - LRD degrades QoS for large queue sizes (beyond some threshold)
  - **but** the threshold depends on the considered QoS metric (loss rate vs mean load)
- Questionable with TCP flows: [Park, 1997] against [Ben Fredj, 2001]
  - LRD has contradictory effects on QoS metrics depending on:

with slow start      without slow start

Delay	↘	↗
loss rate	↘	→
mean throughput	→	↗

- Heavy tailed distributions (i.e LRD) can favour QoS for large flows

# LRD impact on QoS: a brief (experimental) outlook

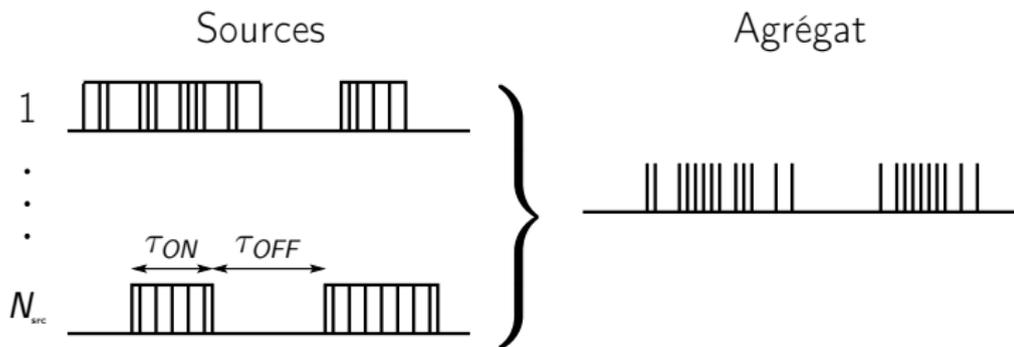
The situation is complex. . .

- Negative on finite queues with UDP flows [cf. Mandjes, 2004 (infinite queues)]
  - LRD degrades QoS for large queue sizes (beyond some threshold)
  - **but** the threshold depends on the considered QoS metric (loss rate vs mean load)
- Questionable with TCP flows: [Park, 1997] against [Ben Fredj, 2001]
  - LRD has contradictory effects on QoS metrics depending on:

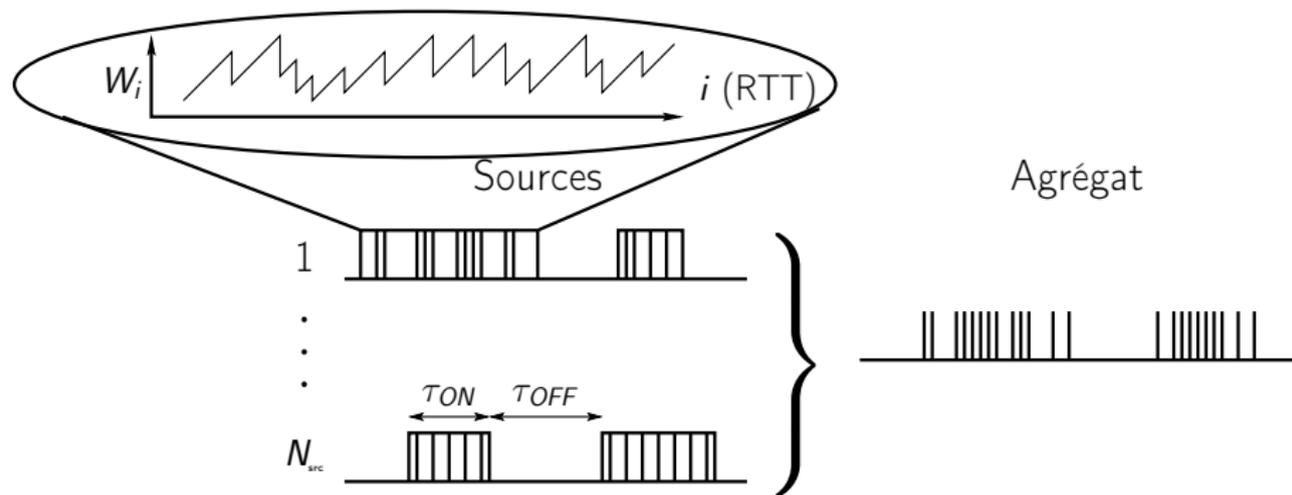
	with slow start	without slow start
Delay	↘	↗
loss rate	↘	→
mean throughput	→	↗

- Heavy tailed distributions (i.e LRD) can favour QoS for large flows
- **But in general, QoS is a complex function of multiple variables**

## Second level of description : single TCP source traffic

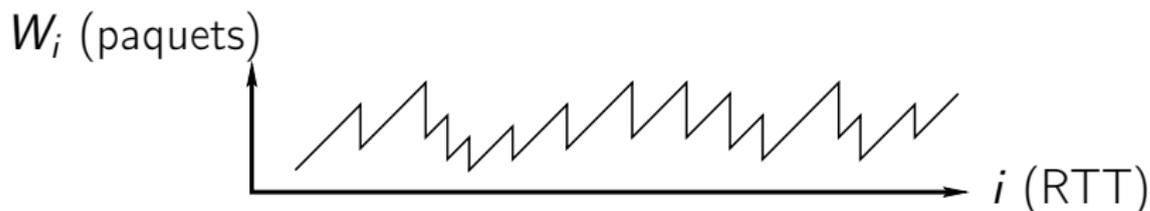


## Second level of description : single TCP source traffic



- single TCP source traffic detail
  - Long-lived flow  $\rightarrow$  stationary regime
- $\Rightarrow$  How to characterize the congestion window evolution?

# Markov model

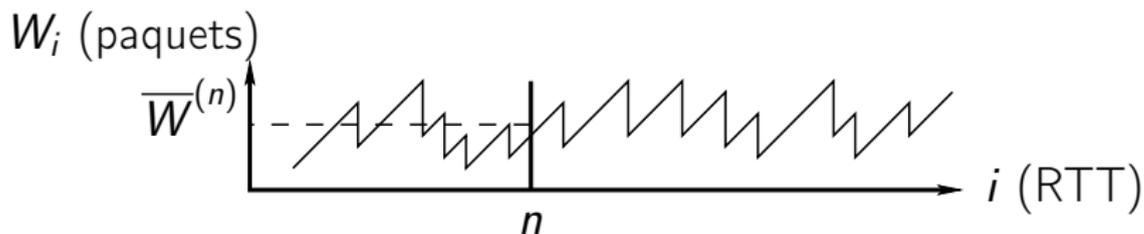


- long-lived flow stationary regime: AIMD
- model:  $(W_i)_{i \geq 1}$  finite Markov chain (irreducible, aperiodic), transition matrix  $Q$  :

$$\begin{cases} Q_{w, \min(w+1, w_{\max})} & = & 1 - p(w), \\ Q_{w, \max(\lfloor w/2 \rfloor, 1)} & = & p(w). \end{cases}$$

- $p(\cdot)$  loss probability of at least one packet, only depends on the current congestion window (hyp.)
- Example: [Padhye, 1998] Bernoulli loss:  $p(w) = 1 - (1 - p_{pkt})^w$

# Almost sure mean throughput



- mean throughput at scale  $n$  (RTT):  $\overline{W}^{(n)} = \frac{\sum_{i=1}^n W_i}{n}$

Ergodic Birkhoff theorem (1931): almost sure mean

For *almost all realisation*, the mean throughput at scale  $n$  converges towards a value corresponding to the expectation of the *invariant distribution*:

$$\overline{W}^{(n)} \xrightarrow[n \rightarrow \infty]{p.s.} \overline{W}^{(\infty)} = \mathbb{E}\{W_i\}$$

- Example: [Padhye, 1998],  $\overline{W}^{(\infty)} \underset{\rho_{pkt} \rightarrow 0}{\sim} \sqrt{\frac{3}{2\rho_{pkt}}}$  (RTT=1, MSS=1)

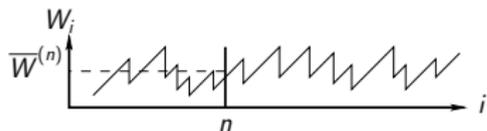
# Throughput variability: Large Deviations

- $\overline{W}^{(n)} \simeq \alpha \neq \overline{W}^{(\infty)}$  Rare events

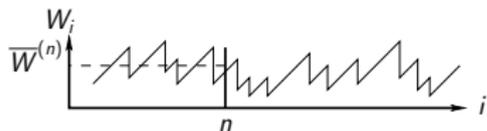
Large Deviations theorem (Ellis, 84)

$$\mathbb{P}(\overline{W}^{(n)} \simeq \alpha) \underset{n \rightarrow \infty}{\sim} \exp(n \cdot f(\alpha))$$

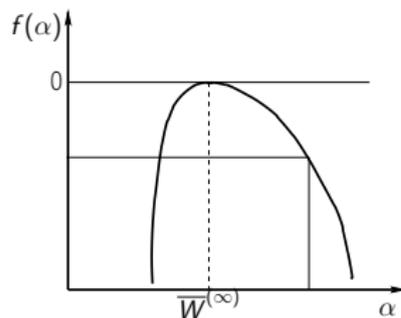
- $f(\alpha)$  Large Deviation spectrum
- Scale invariant quantity



•  
•

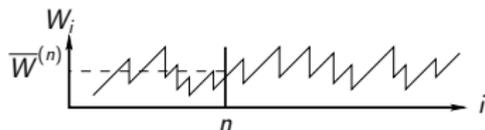


•  
•  
•

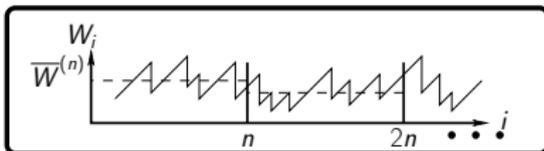


# Throughput variability: Large Deviations

- $\overline{W}^{(n)} \simeq \alpha \neq \overline{W}^{(\infty)}$  Rare events



⋮

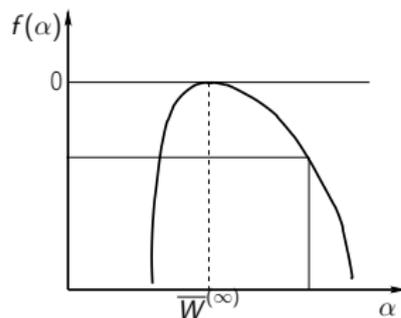


⋮

Large Deviations theorem (Ellis, 84)

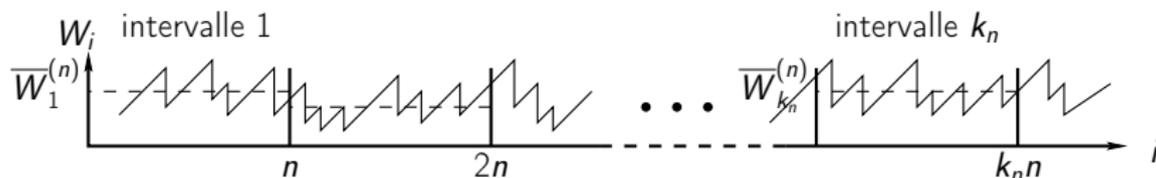
$$\mathbb{P}(\overline{W}^{(n)} \simeq \alpha) \underset{n \rightarrow \infty}{\sim} \exp(n \cdot f(\alpha))$$

- $f(\alpha)$  Large Deviation spectrum
- Scale invariant quantity



⇒ Does a similar theorem exist for a single realization?

# Large Deviation on almost all realizations



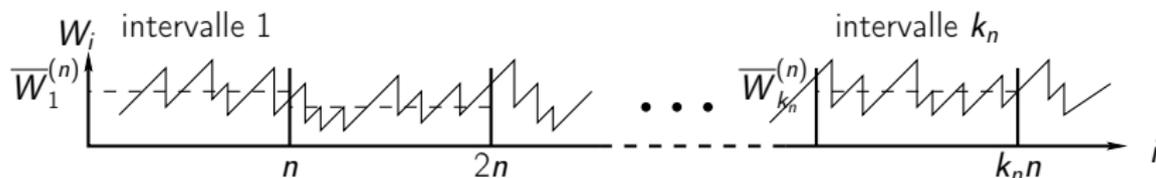
Large Deviation theorem on almost all realisations (Loiseau et al., 2010)

For a given  $\alpha$ , if  $k_n \geq e^{nR(\alpha)}$ , then a.s.

$$\frac{\#\{j \in \{1, \dots, k_n\} : \overline{W}_j^{(n)} \simeq \alpha\}}{k_n} \underset{n \rightarrow \infty}{\sim} \exp(n \cdot f(\alpha))$$

- “Price to pay”: exponential increase of the number of intervals
  - Finite realization (of size  $N$ ):  $nk_n = N$
- $\Rightarrow [\alpha_{\min}(n), \alpha_{\max}(n)]$  support of observable spectrum at scale  $n$

# Large Deviation on almost all realizations



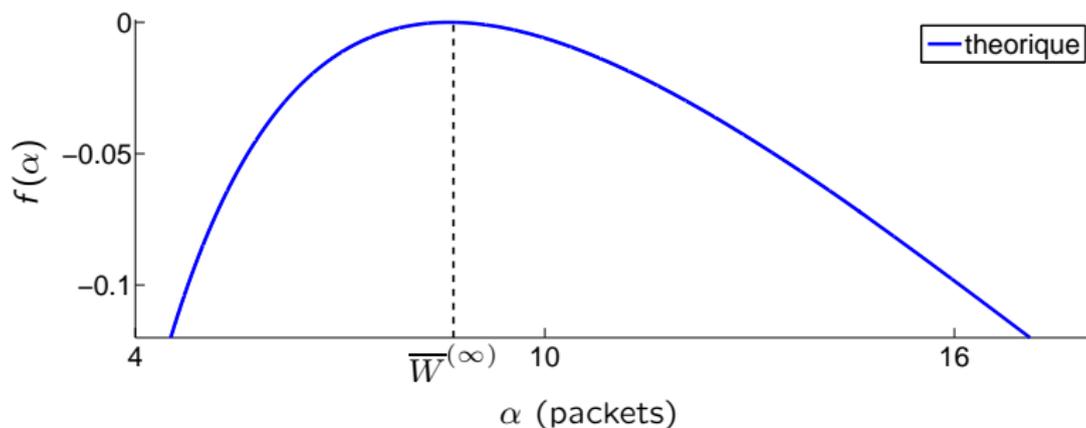
Large Deviation theorem on almost all realisations (Loiseau et al., 2010)

For a given  $\alpha$ , if  $k_n \geq e^{nR(\alpha)}$ , then a.s.

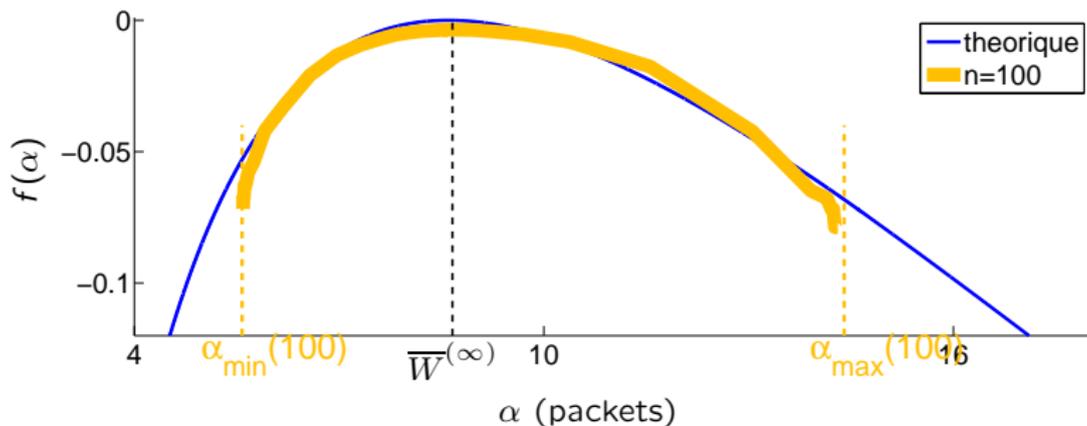
$$\frac{\#\{j \in \{1, \dots, k_n\} : \overline{W}_j^{(n)} \simeq \alpha\}}{k_n} \underset{n \rightarrow \infty}{\sim} \exp(n \cdot f(\alpha))$$

- “Price to pay”: exponential increase of the number of intervals
  - Finite realization (of size  $N$ ):  $nk_n = N$
- $\Rightarrow [\alpha_{\min}(n), \alpha_{\max}(n)]$  support of observable spectrum at scale  $n$
- Theory:  $p(\cdot) \rightarrow Q \rightarrow f(\alpha), R(\alpha), \alpha_{\min}, \alpha_{\max}$
  - Practice:  $(W_i)_{i \leq N} \rightarrow$  observed distribution

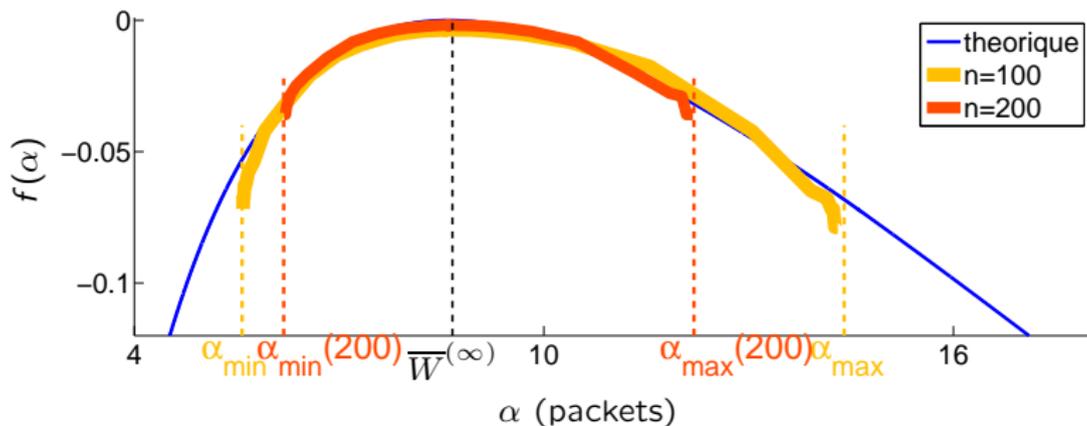
Results: example of Bernoulli losses ( $p_{\text{pkt}} = 0.02$ )



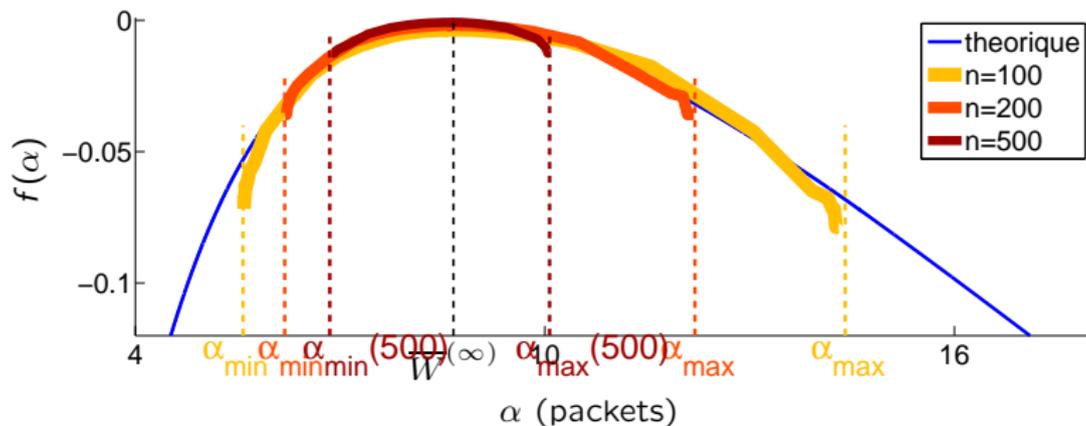
# Results: example of Bernoulli losses ( $p_{pkt} = 0.02$ )



- Apex: almost sure mean: 8.6 packets (Padhye:  $\sqrt{\frac{3}{2p_{pkt}}} = 8.66$ )

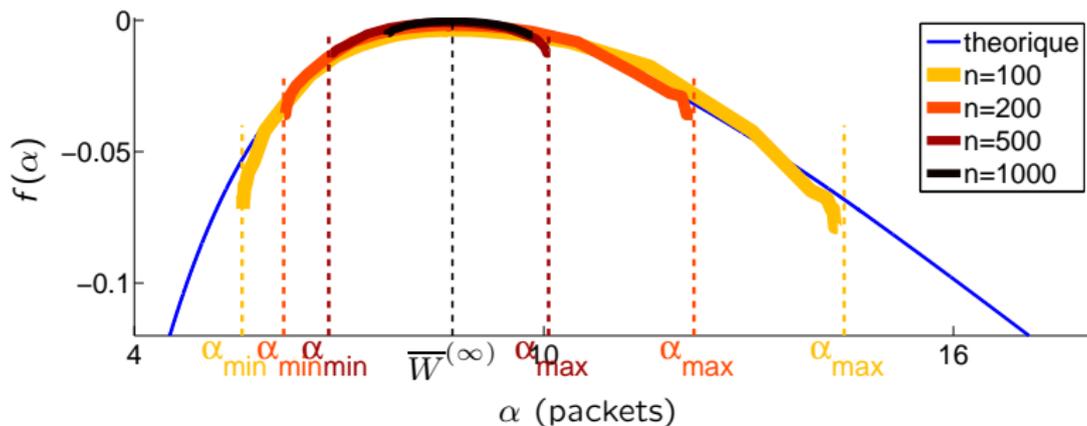
Results: example of Bernoulli losses ( $p_{\text{pkt}} = 0.02$ )

- Apex: almost sure mean: 8.6 packets (Padhye:  $\sqrt{\frac{3}{2p_{\text{pkt}}}} = 8.66$ )

Results: example of Bernoulli losses ( $p_{\text{pkt}} = 0.02$ )

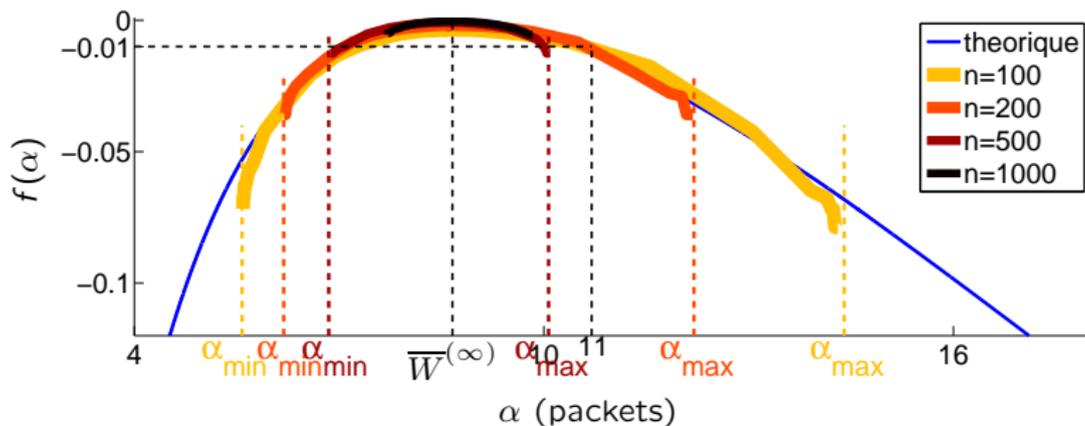
- Apex: almost sure mean: 8.6 packets (Padhye:  $\sqrt{\frac{3}{2p_{\text{pkt}}}} = 8.66$ )

# Results: example of Bernoulli losses ( $p_{pkt} = 0.02$ )



- Apex: almost sure mean: 8.6 packets (Padhye:  $\sqrt{\frac{3}{2p_{pkt}}} = 8.66$ )
- Superimposition at different scales  $\rightarrow$  **scale invariance**

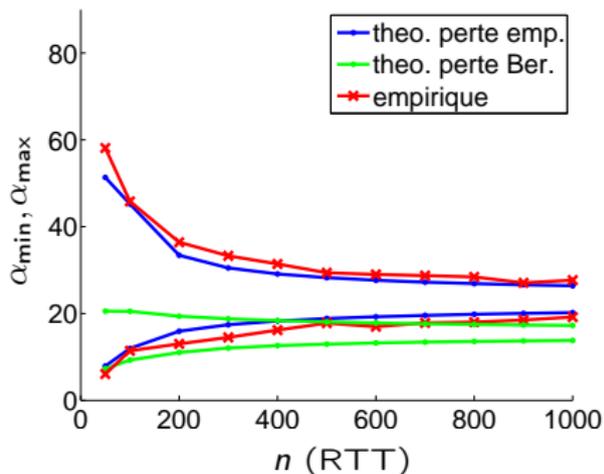
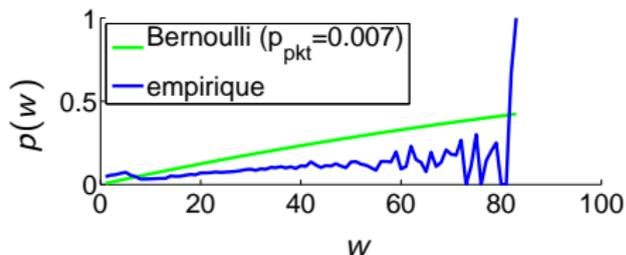
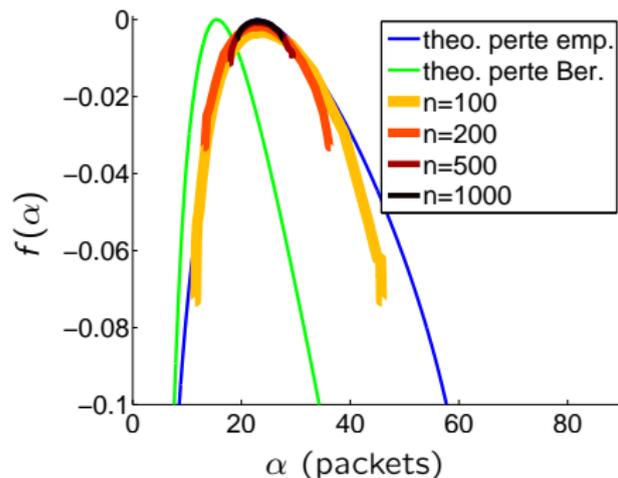
# Results: example of Bernoulli losses ( $p_{pkt} = 0.02$ )



- Apex: almost sure mean: 8.6 packets (Padhye:  $\sqrt{\frac{3}{2p_{pkt}}} = 8.66$ )
  - Superimposition at different scales → **scale invariance**
  - beyond  $n = 100$ : variability
    - $n = 100$ , portion of intervals with mean  $\sim 11$ :  $e^{-100 \times 0.01} = 0.37$
    - $n = 200$ , portion of intervals with mean  $\sim 11$ :  $e^{-200 \times 0.01} = 0.14$
- ⇒ **More accurate information than the almost sure mean**

## Results II: case of a long-lived flow

- losses: not Bernoulli
- empirical losses



## Two important assets for Large Deviations Utility

General result ( "Large deviations for the local fluctuations of random walks", J. Barral, P. Loiseau, *Stochastic Processes and their Applications*, 2011)

A wide class of processes (stationary & mixing) verifies an *empirical large deviation principle*. In particular, this results holds true any time series that can reliably be modelled by an *irreducible, aperiodic Markov process*.

Theorem ( "On the estimation of the Large Deviations spectrum", J. Barral, P. G., *J. stat. Phys.*, 2011)

We derived a *consistent estimator of the large deviation spectrum* from a finite size time series (observation samples). We proved *convergence* on mathematical objects with scale invariance properties (multifractal measures and processes).

Empirical estimation from a finite length trace

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo highly time-varying (elastic) workloads (e.g. Buzz demand in a VoD system)

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

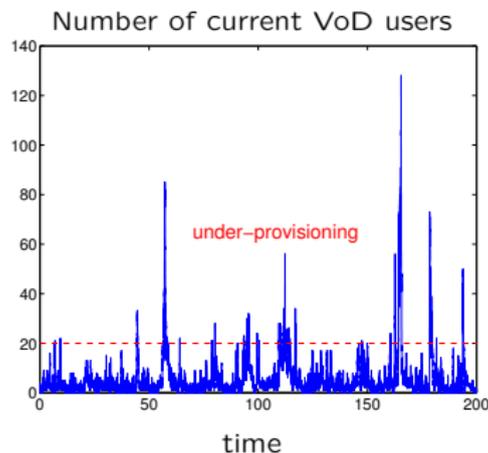
- Context** Applications that undergo highly time-varying (elastic) workloads (e.g. Buzz demand in a VoD system)
- Goal** Dynamic resource allocation yielding a good compromise between capex and opex costs

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo highly time-varying (elastic) workloads (e.g. Buzz demand in a VoD system)

**Goal** Dynamic resource allocation yielding a good compromise between capex and opex costs

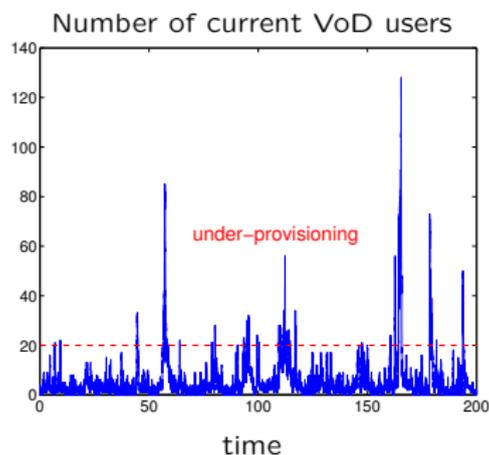


# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo highly time-varying (elastic) workloads (e.g. Buzz demand in a VoD system)

**Goal** Dynamic resource allocation yielding a good compromise between capex and opex costs



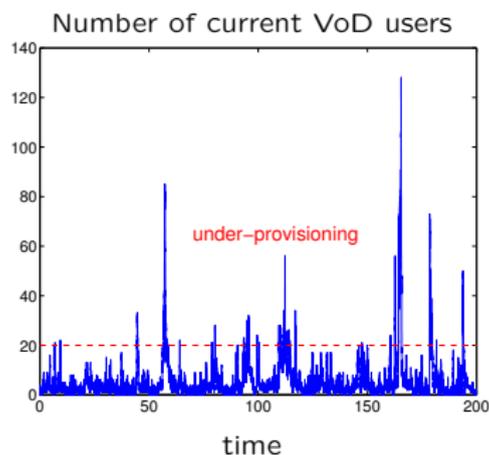
**Approach** Combine the three ingredients:

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo **highly time-varying (elastic) workloads** (e.g. Buzz demand in a VoD system)

**Goal** **Dynamic resource allocation** yielding a good compromise between **capex** and **opex** costs



**Approach** Combine the three ingredients:

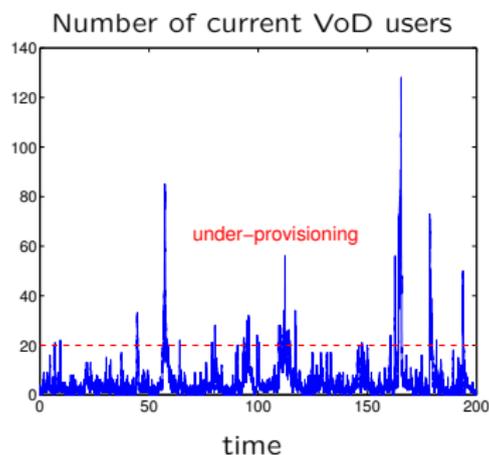
- A **sensible** (epidemic) model to catch the **burstiness** and the **dynamics** of the workload

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo **highly time-varying (elastic) workloads** (e.g. Buzz demand in a VoD system)

**Goal** **Dynamic resource allocation** yielding a good compromise between **capex** and **opex** costs



**Approach** Combine the three ingredients:

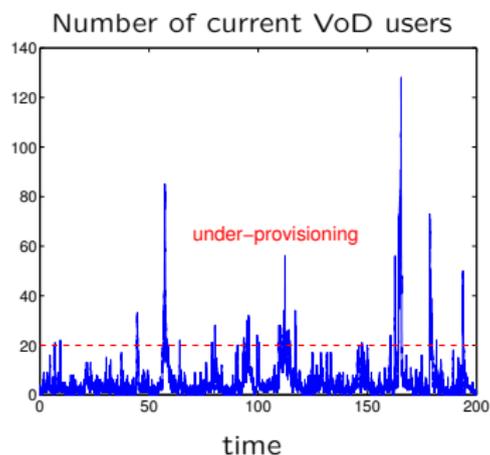
- A **sensible** (epidemic) model to catch the **burstiness** and the **dynamics** of the workload
- A (Markov) model that verifies a **large deviations principle**

# Probabilistic Resources Allocation Based on a LDP

## Workload Volatility

**Context** Applications that undergo **highly time-varying (elastic) workloads** (e.g. Buzz demand in a VoD system)

**Goal** **Dynamic resource allocation** yielding a good compromise between **capex** and **opex** costs

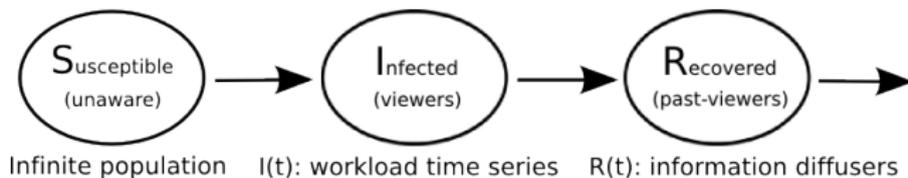


**Approach** Combine the three ingredients:

- A **sensible** (epidemic) model to catch the **burstiness** and the **dynamics** of the workload
- A (Markov) model that verifies a **large deviations principle**
- A **probabilistic** management policy based on the large deviation characterisation

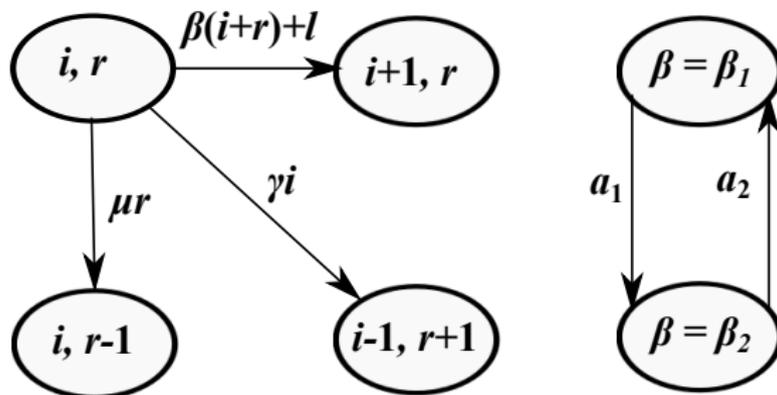
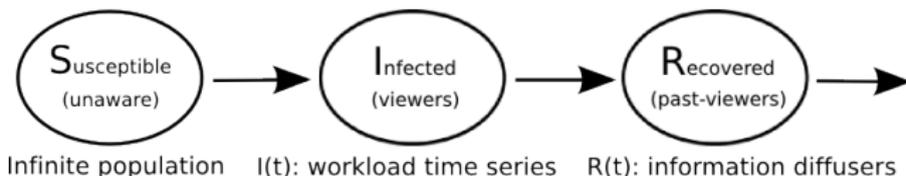
# An epidemic based model for volatile workload

A hidden state Markov process with memory effect [IEICE 2012, TRAC 2013]



# An epidemic based model for volatile workload

A hidden state Markov process with memory effect [IEICE 2012, TRAC 2013]

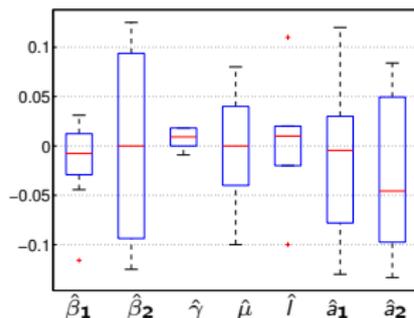


$i$ : current # of viewers /  $r$ : current # of infected

# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

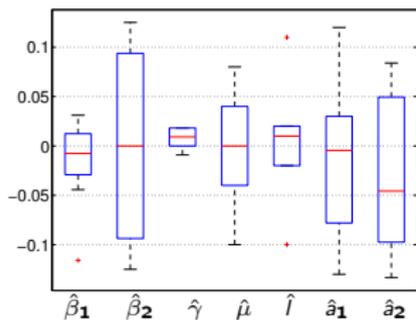
Param. estimation precision



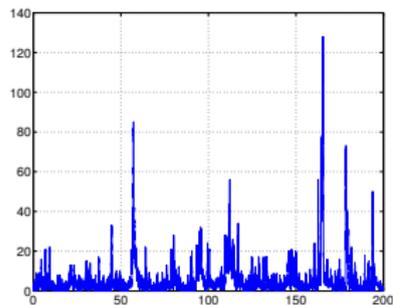
# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

Param. estimation precision



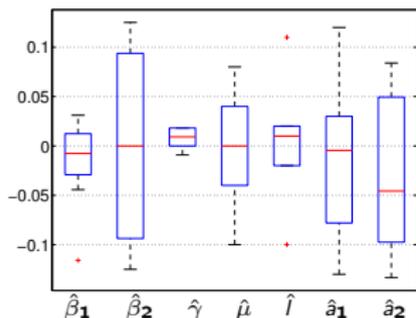
VoD workload - trace I



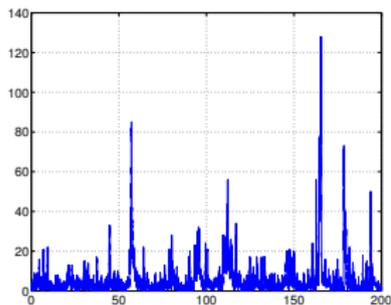
# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

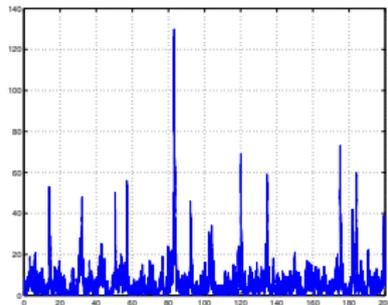
Param. estimation precision



VoD workload - trace I



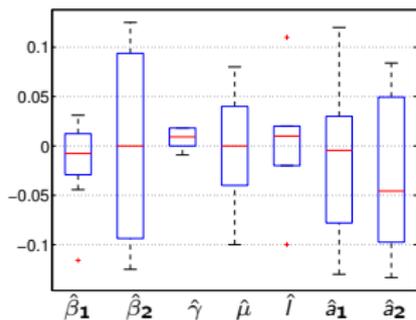
Proposed Model



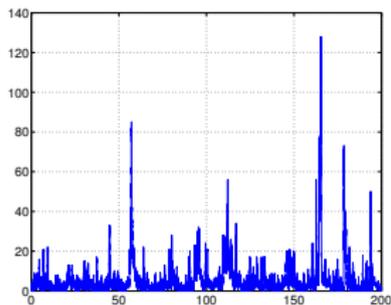
# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

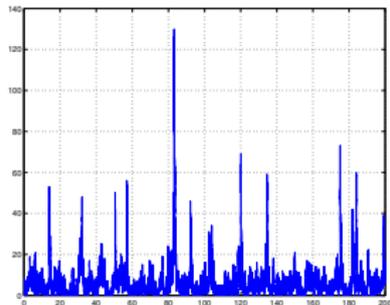
Param. estimation precision



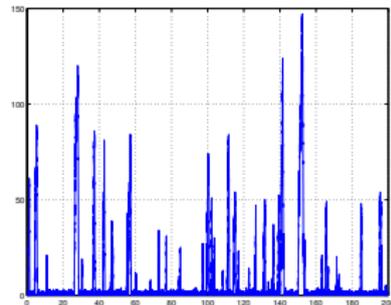
VoD workload - trace I



Proposed Model



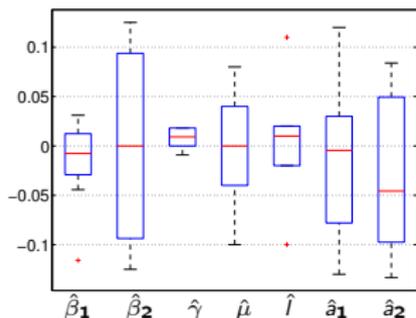
MMPP/M/1



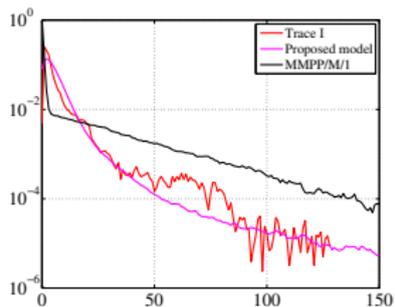
# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

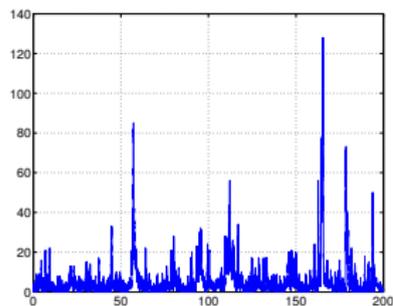
Param. estimation precision



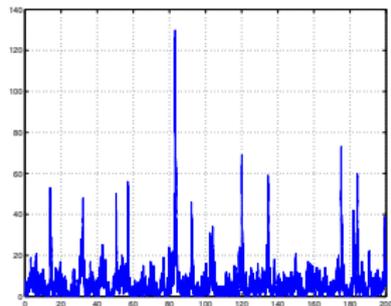
Steady state distribution



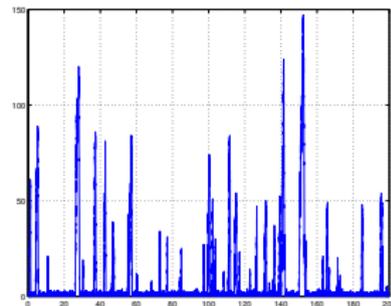
VoD workload - trace I



Proposed Model



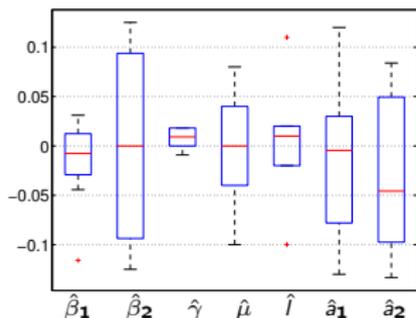
MMPP/M/1



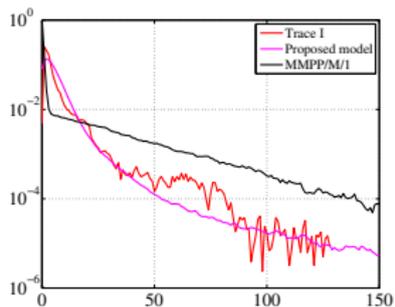
# Model identification and evaluation

A MCMC based estimation procedure for the model's parameters [Gretsi 2013]

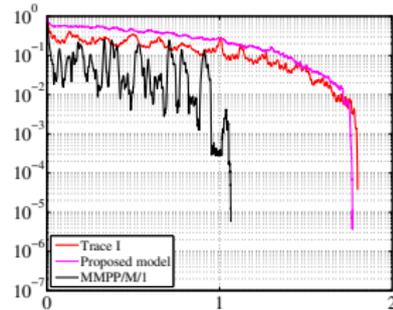
Param. estimation precision



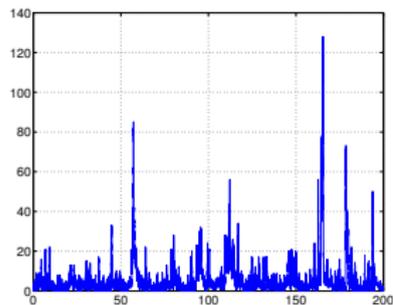
Steady state distribution



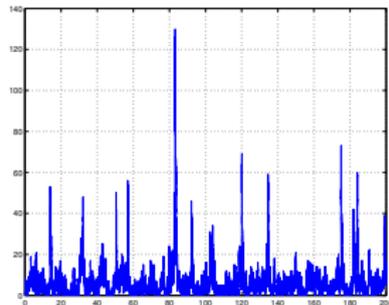
Autocorrelation function



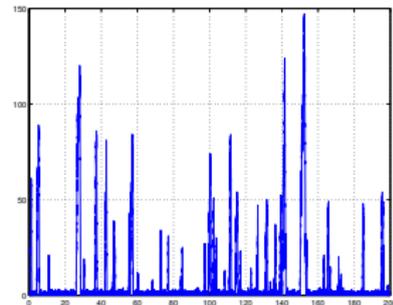
VoD workload - trace I



Proposed Model



MMPP/M/1



# Large Deviations Principle

A process  $I_t$  verifies a large deviations principle:

$$\mathbb{P}\{\langle I_t \rangle_\tau \in [\alpha - \varepsilon_\tau, \alpha + \varepsilon_\tau]\} \sim \exp(\tau \cdot f(\alpha)), \quad \tau \rightarrow \infty$$

$\tau$  : average time scale

$f(\alpha)$  : large deviations spectrum of  $I_t$

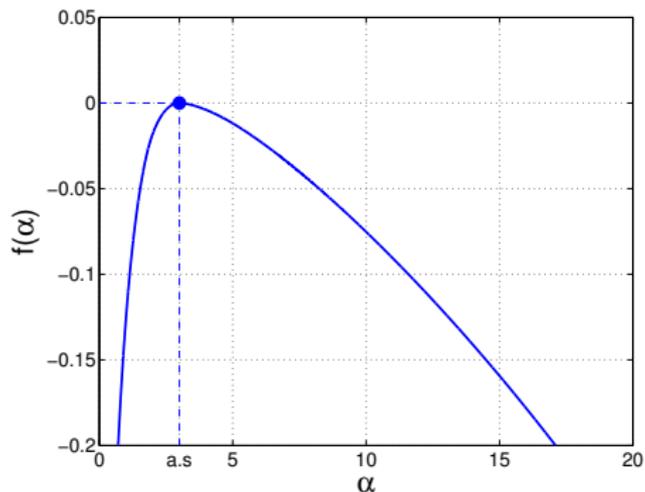
# Large Deviations Principle

A process  $I_t$  verifies a large deviations principle:

$$\mathbb{P}\{\langle I_t \rangle_\tau \in [\alpha - \varepsilon_\tau, \alpha + \varepsilon_\tau]\} \sim \exp(\tau \cdot f(\alpha)), \quad \tau \rightarrow \infty$$

$\tau$  : average time scale

$f(\alpha)$  : large deviations spectrum of  $I_t$



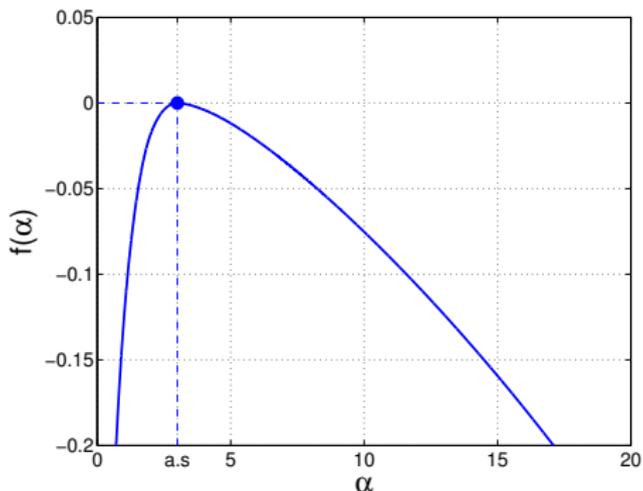
# Large Deviations Principle

A process  $I_t$  verifies a large deviations principle:

$$\mathbb{P}\{\langle I_t \rangle_\tau \in [\alpha - \varepsilon_\tau, \alpha + \varepsilon_\tau]\} \sim \exp(\tau \cdot f(\alpha)), \quad \tau \rightarrow \infty$$

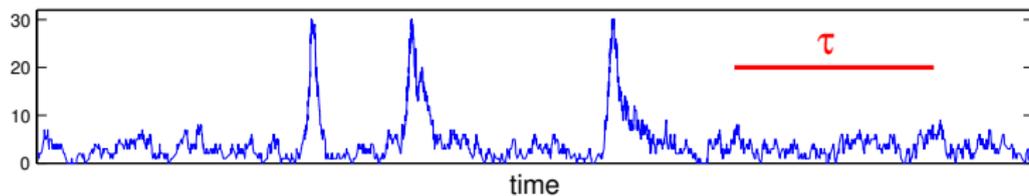
$\tau$  : average time scale

$f(\alpha)$  : large deviations spectrum of  $I_t$

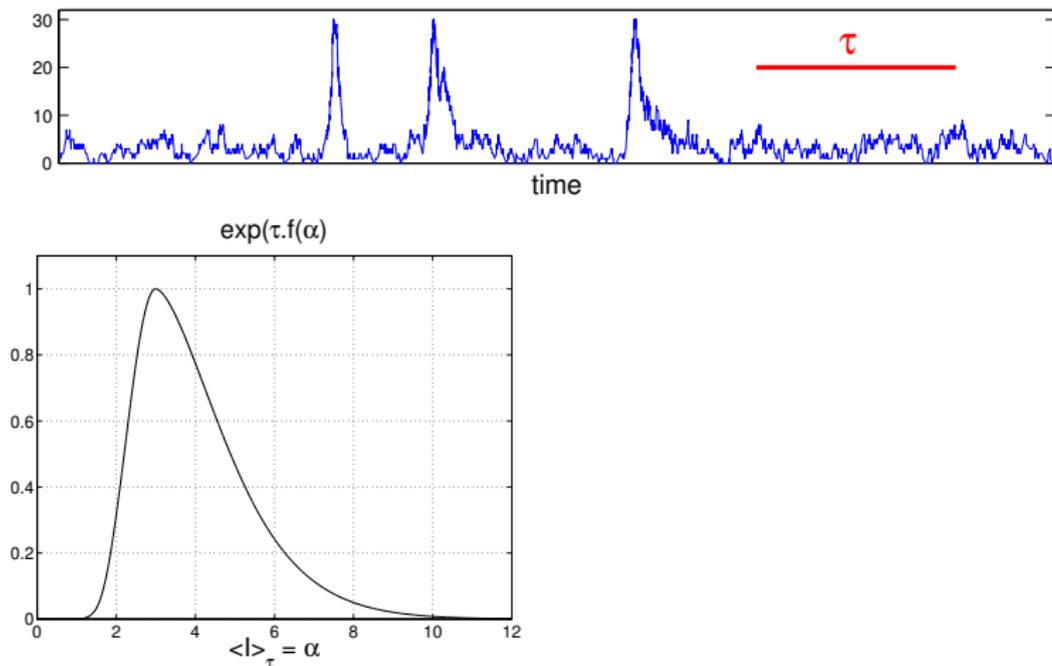


"Dynamic" implies time scale: a notion that is explicit in large deviations principle

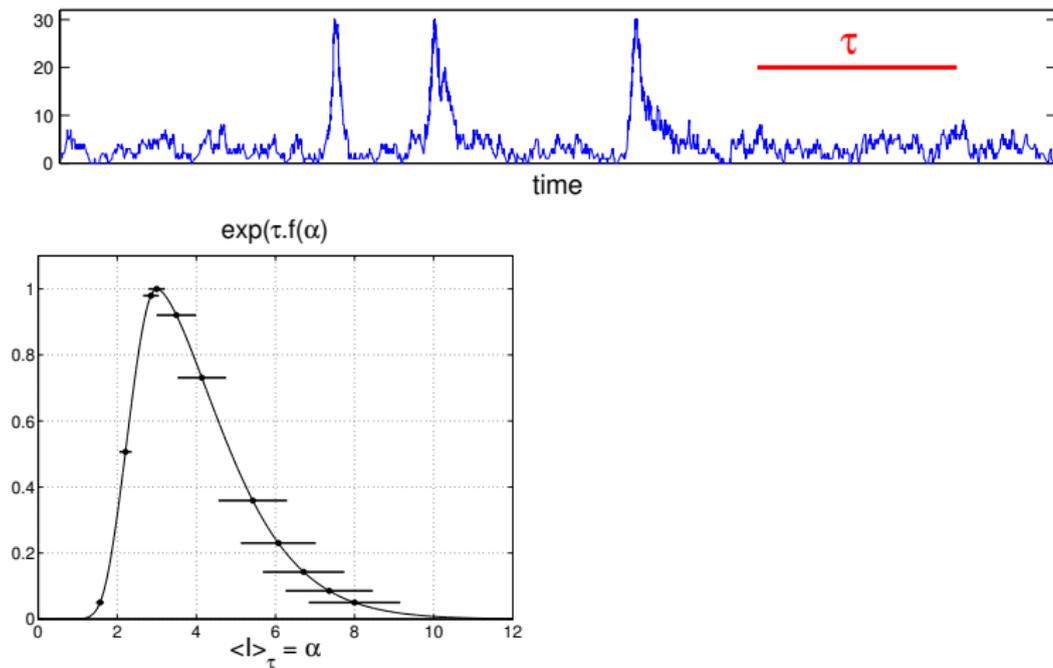
# Overflow propability



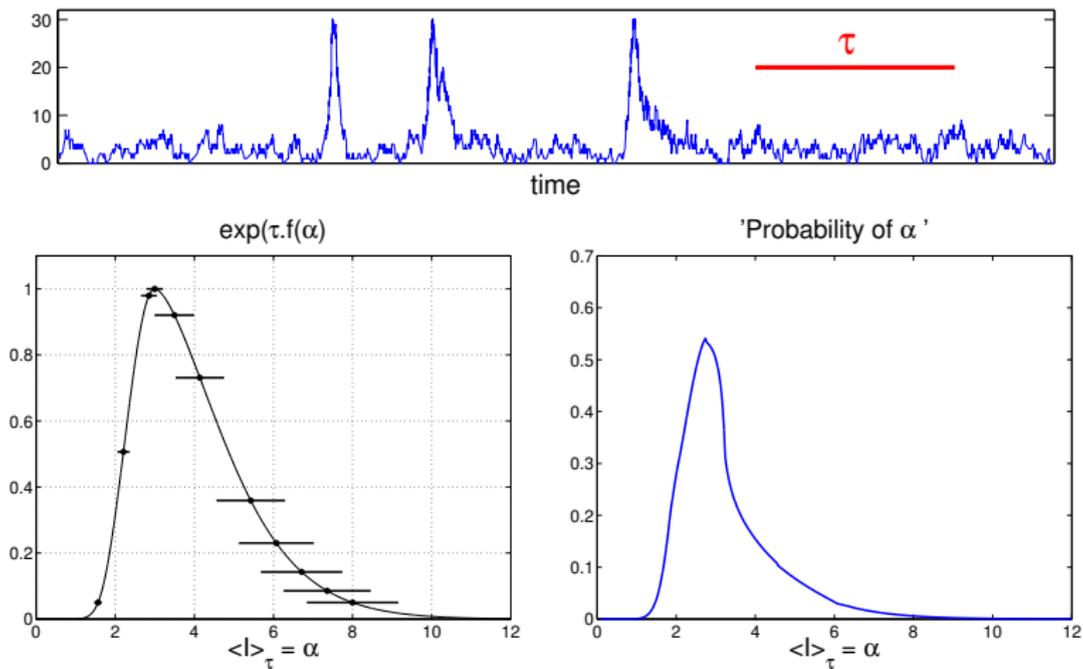
# Overflow probability



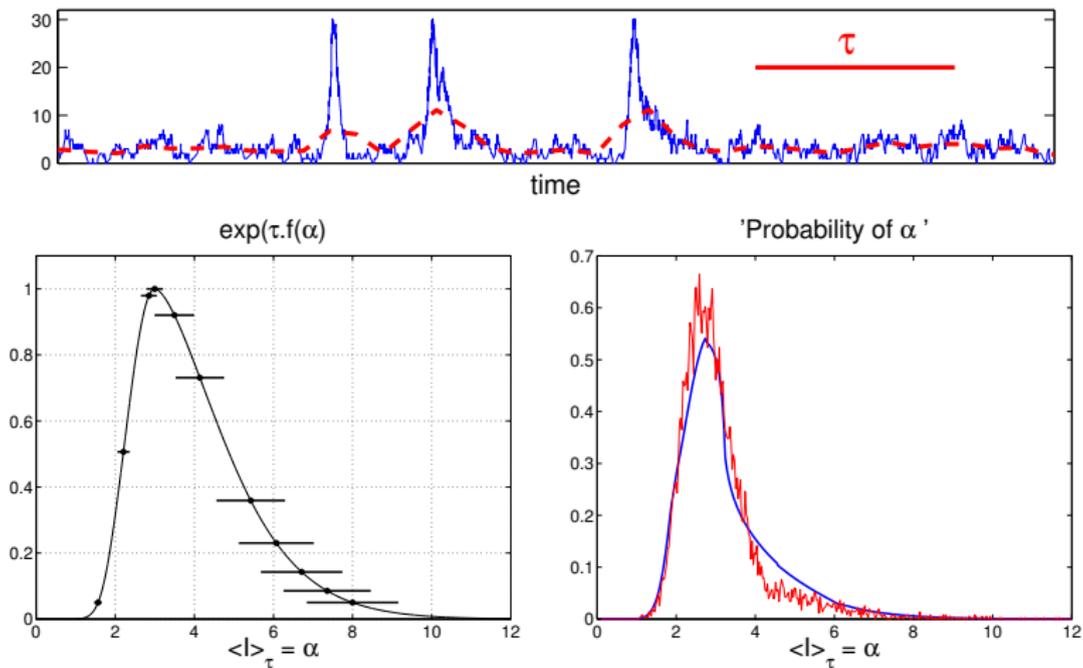
# Overflow probability



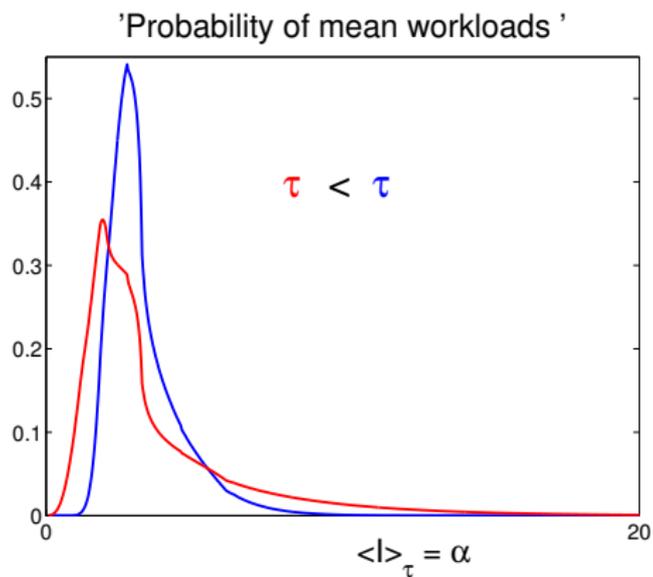
# Overflow probability



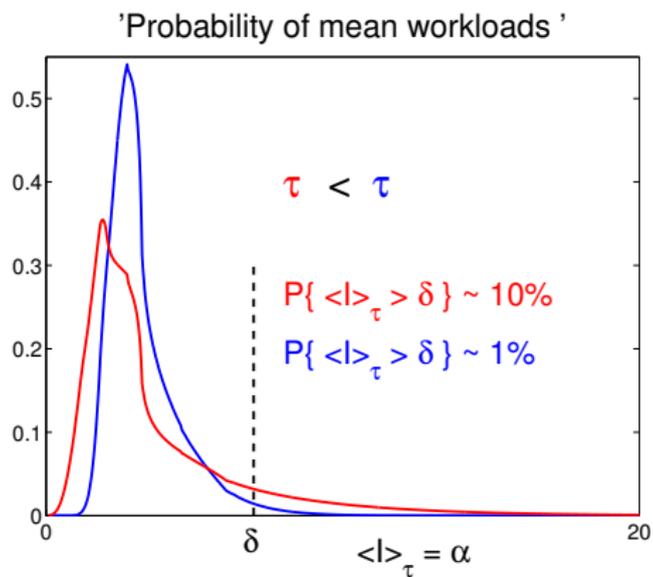
# Overflow probability



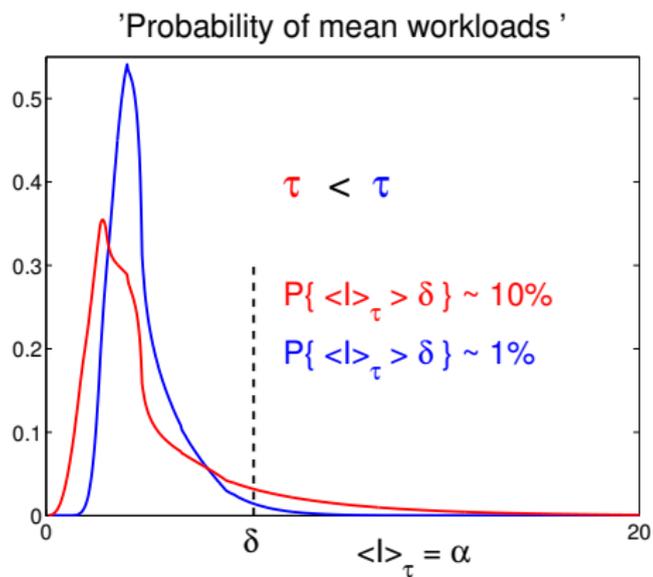
# Probabilistic resource provisioning



# Probabilistic resource provisioning

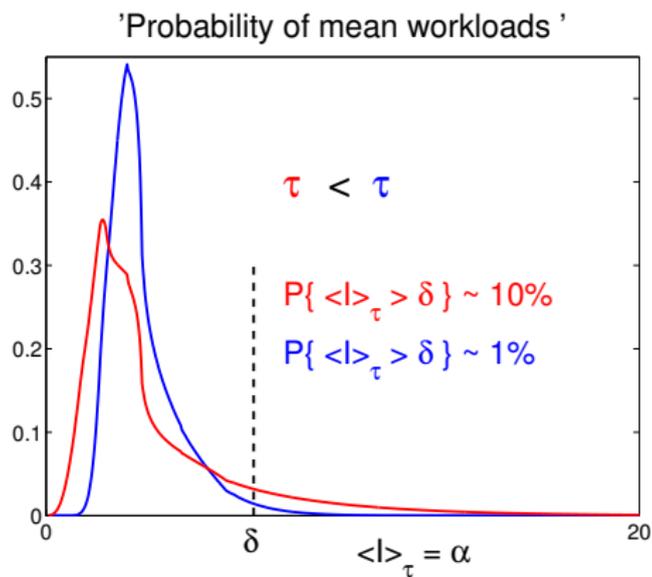


# Probabilistic resource provisioning



→ Resource provisioning based on a **time scale** dependent performance evaluation

# Probabilistic resource provisioning



→ Resource provisioning based on a **time scale** dependent performance evaluation

→ **Dynamic management**

# Optimal reactive time scale for reconfiguration

# Optimal reactive time scale for reconfiguration

**Reactivity scale for reconfiguring resource allocation is a compromise between:**

- the level of congestion (or losses) yielding tolerable performance degradation
- the affordable price for a frequent reconfiguration of infrastructures

# Optimal reactive time scale for reconfiguration

**Reactivity scale for reconfiguring resource allocation is a compromise between:**

- the level of congestion (or losses) yielding tolerable performance degradation
- the affordable price for a frequent reconfiguration of infrastructures

**Assume admissible bounds for these 2 competing factors:**

$\alpha^* > \alpha_{a.s.}$  beyond, it is mandatory (or profitable) reallocating resources

← capex performance concern

$\sigma^*$  acceptable probability of occurrence of overflows

← opex cost

# Optimal reactive time scale for reconfiguration

**Reactivity scale for reconfiguring resource allocation is a compromise between:**

- the level of congestion (or losses) yielding tolerable performance degradation
- the affordable price for a frequent reconfiguration of infrastructures

**Assume admissible bounds for these 2 competing factors:**

$\alpha^* > \alpha_{a.s.}$  beyond, it is mandatory (or profitable) reallocating resources  
← capex performance concern

$\sigma^*$  acceptable probability of occurrence of overflows  
← opex cost

**and  $f(\alpha)$  is identifiable**

# Optimal reactive time scale for reconfiguration

**Reactivity scale for reconfiguring resource allocation is a compromise between:**

- the level of congestion (or losses) yielding tolerable performance degradation
- the affordable price for a frequent reconfiguration of infrastructures

**Assume admissible bounds for these 2 competing factors:**

$\alpha^* > \alpha_{\text{a.s.}}$  beyond, it is mandatory (or profitable) reallocating resources  
 ← capex performance concern

$\sigma^*$  acceptable probability of occurrence of overflows  
 ← opex cost

**and**  $f(\alpha)$  is identifiable

**Optimal reconfiguration time scale for dynamic resource provisioning:**

$$\tau^* : \Pr\{\langle I \rangle_{\tau^*} \geq \alpha^*\} \approx \int_{\alpha^*}^{\infty} P_{\tau^*}(\alpha) d\alpha > \sigma^*$$

# Elastic link capacity dimensioning

# Elastic link capacity dimensioning

## The Service Level Agreement fixes:

- ...
- an admissible level of losses due to link congestion

# Elastic link capacity dimensioning

The Service Level Agreement fixes:

- ...
- an admissible level of losses due to link congestion

Assume  $f(\alpha)$  is identifiable

# Elastic link capacity dimensioning

The Service Level Agreement fixes:

- ...
- an admissible level of losses due to link congestion

Assume  $f(\alpha)$  is identifiable

$C_0 = \alpha_{a.s.}$  The **dedicated** link capacity (nominal functioning)

# Elastic link capacity dimensioning

The Service Level Agreement fixes:

- ...
- an admissible level of losses due to link congestion

Assume  $f(\alpha)$  is identifiable

$C_0 = \alpha_{a.s.}$  The **dedicated** link capacity (nominal functioning)

$\tilde{C}_{\tau_{\min}}$  The **shared** bandwidth needed to absorb bursty overflows, while guaranteeing **QoS** (loss rate) conformed to SLA:

$$\tilde{C}_{\tau_{\min}} = \int_{\alpha_{a.s.}}^{\infty} (\alpha - \alpha_{a.s.}) P_{\tau_{\min}}(\alpha) d\alpha$$

# Elastic link capacity dimensioning

The Service Level Agreement fixes:

- ...
- an admissible level of losses due to link congestion

Assume  $f(\alpha)$  is identifiable

$C_0 = \alpha_{a.s.}$  The **dedicated** link capacity (nominal functioning)

$\tilde{C}_{\tau_{\min}}$  The **shared** bandwidth needed to absorb bursty overflows, while guaranteeing **QoS** (loss rate) conformed to SLA:

$$\tilde{C}_{\tau_{\min}} = \int_{\alpha_{a.s.}}^{\infty} (\alpha - \alpha_{a.s.}) P_{\tau_{\min}}(\alpha) d\alpha$$

$\tau_{\min}$  Determined by the buffer size provisioned to dampen traffic volatility

# Concluding remarks

# Concluding remarks

- Scaling laws
  - Present in many (complex) systems
  - Likely to become ever more ubiquitous (big data sets, heterogeneity, traffic awareness. . .)
  - Impact (on performance) are still little known

# Concluding remarks

## Scaling laws

Present in many (complex) systems

Likely to become ever more ubiquitous (big data sets, heterogeneity, traffic awareness. . .)

Impact (on performance) are still little known

## Large Dev. Princ.

Insufficiently exploited so far

Holds true for a large class of modelling processes

Takes explicitly into account the role of time scale

Conveys information about the dynamics of the process